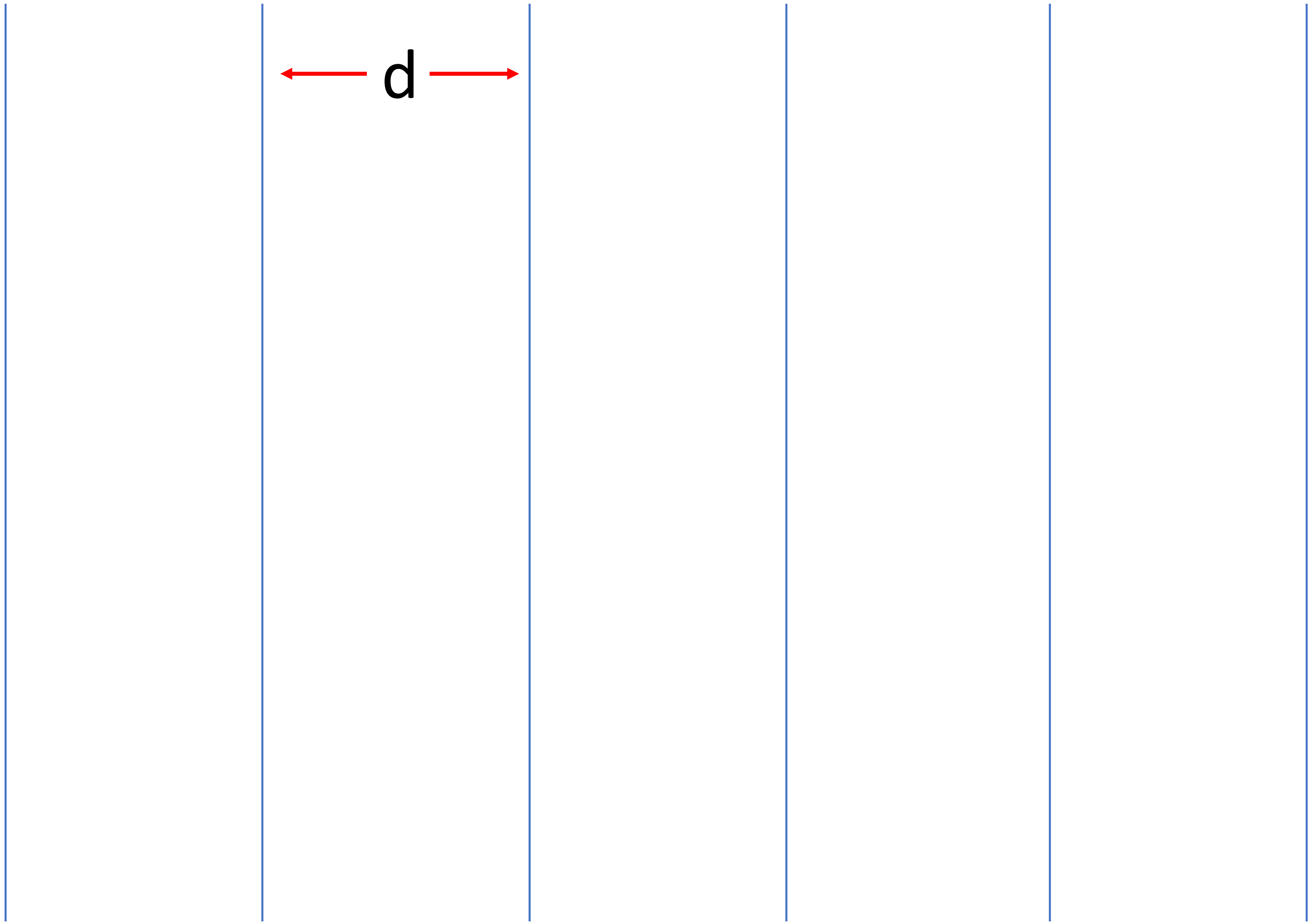
My talk is full of *eels*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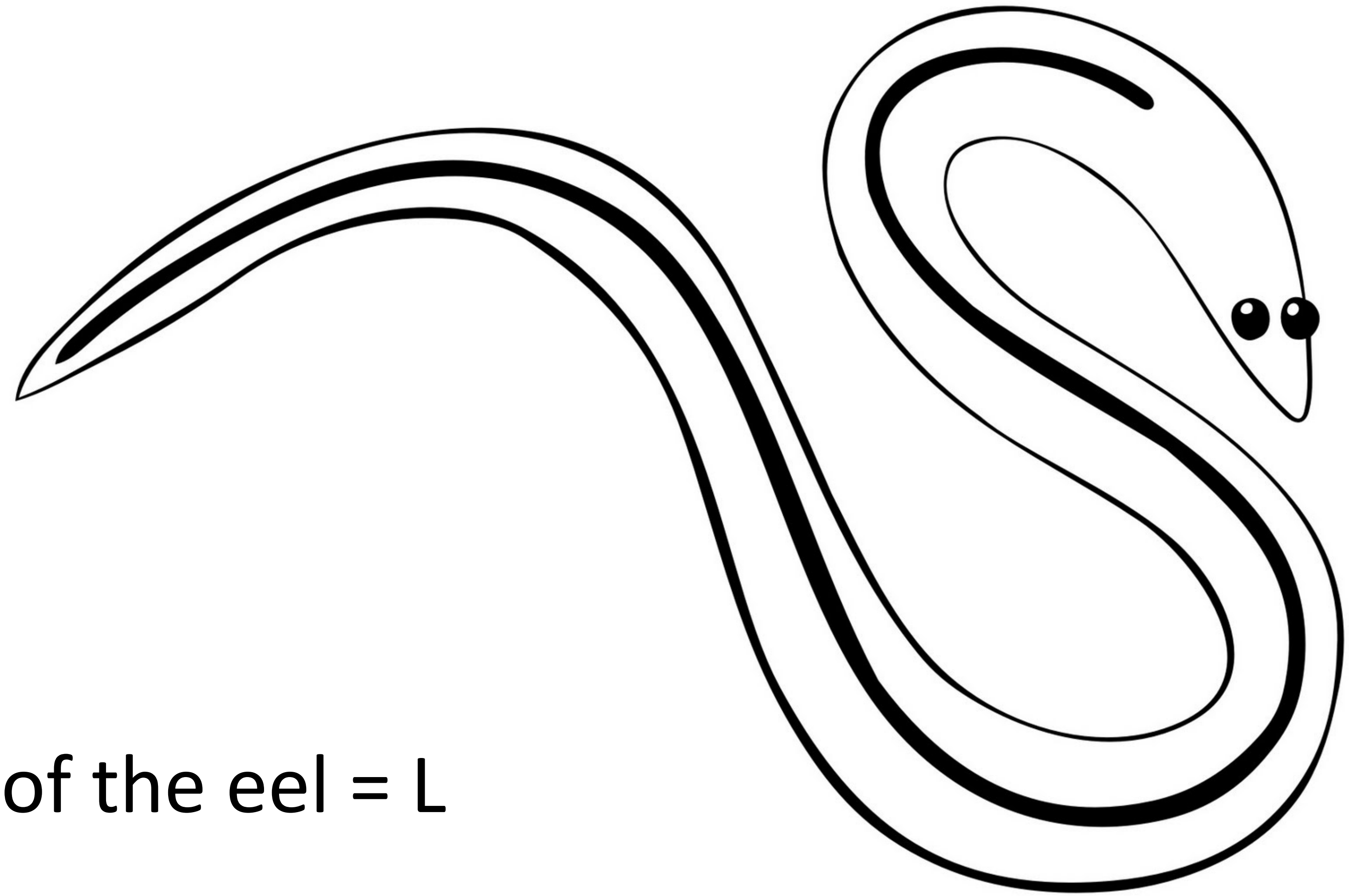
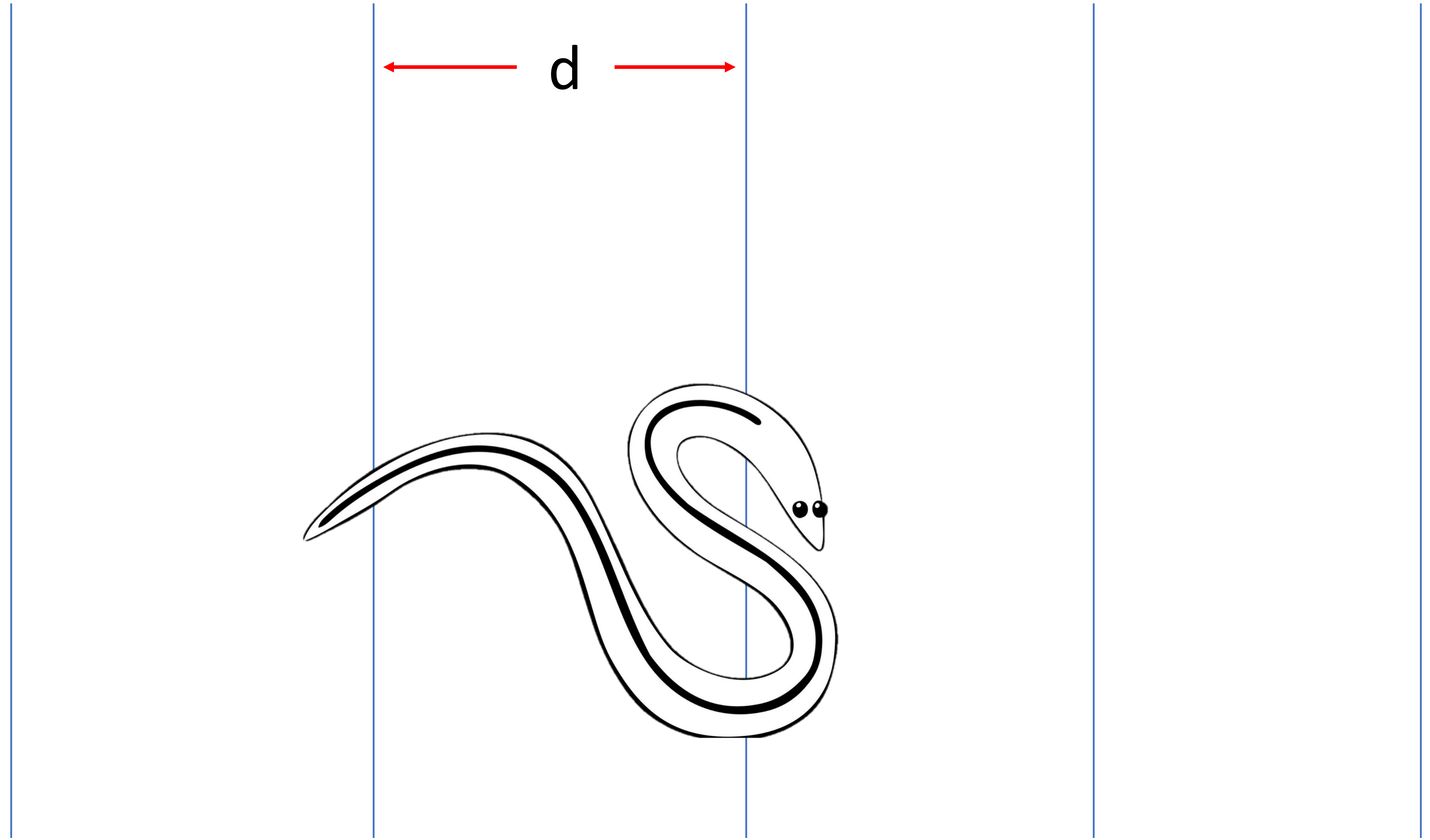


Length of the eel = L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Question:

What is the expected number of crossings?

- First, suppose the eel is straight and L < d

Let

- X: number of crossings after tossing the eel
- p: probability that the eel crosses at least one line

Expectation of X:

E(X) = $\sum_n nP(X = n)$.

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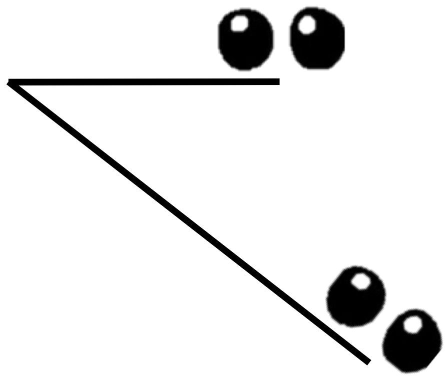
But X cannot be more than 1

$$E(X) = p.$$

Suppose we have another eel of length L’ and we put them together



New length = L + L’

X’: expected number of crossings

p’: probability of at least 1 crossing

Key argument:

The values of X and X' are not independent, but

$$E(X+X') = E(X) + E(X') \text{ !!!}$$

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If $E(X) = f(L)$, then

$$f(L+L') = f(L) + f(L')$$

Then

$$f(L) = cL$$

for some $c>0$. What is the value of c?

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Let’s go back to the original eel:

- L can be larger than d
- Not straight anymore

If the eel is rectifiable, then by approximating with short linear segments,

$$E(X) = cL$$

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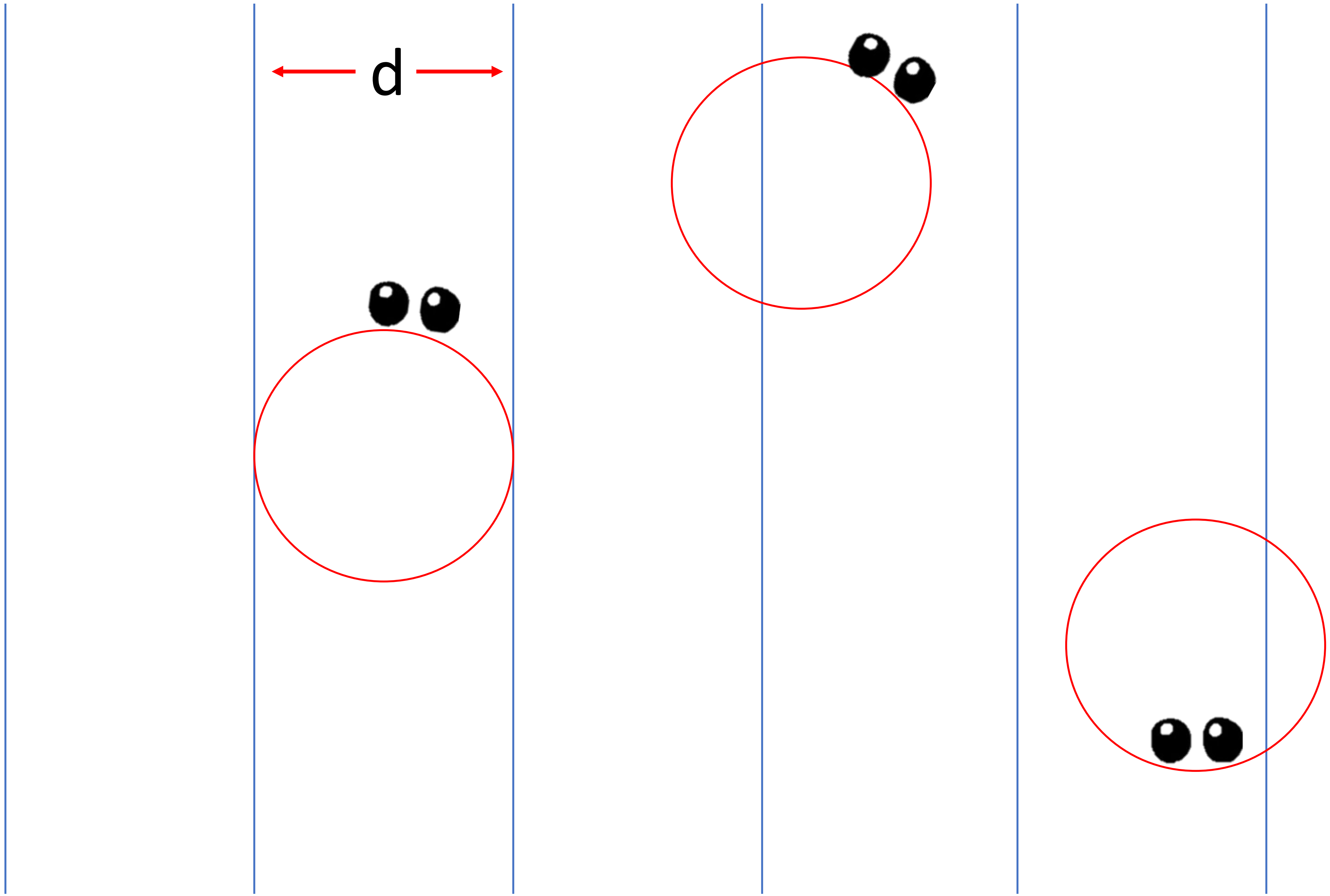
In particular, it holds for a circular eel of diameter d


d

For this eel

$$E(X) = cL = c(d\pi) = 2$$

and so

$$c = \frac{2}{d\pi}.$$

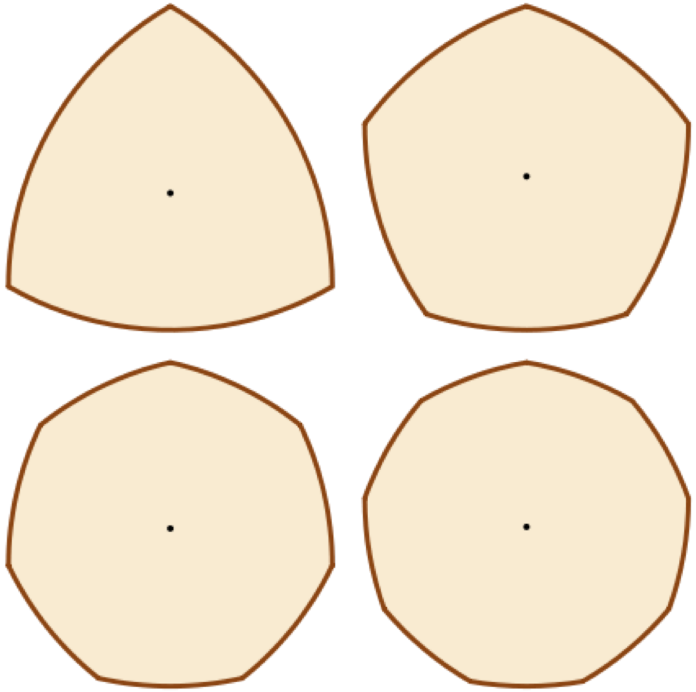
We conclude that for a general eel, the expected number of crossings is

$$E(X) = \frac{2L}{d\pi} \ .$$

Bonus!

Theorem (Barbier's):

Let A be a curve of constant width d, then its perimeter L is dπ.



Proof:

Toss such curve on an array of parallel lines separated by a distance d. The expected number of crossings is

$$E(X) = \frac{2L}{d\pi} = 2$$

Thus

$$L = d\,\pi.$$