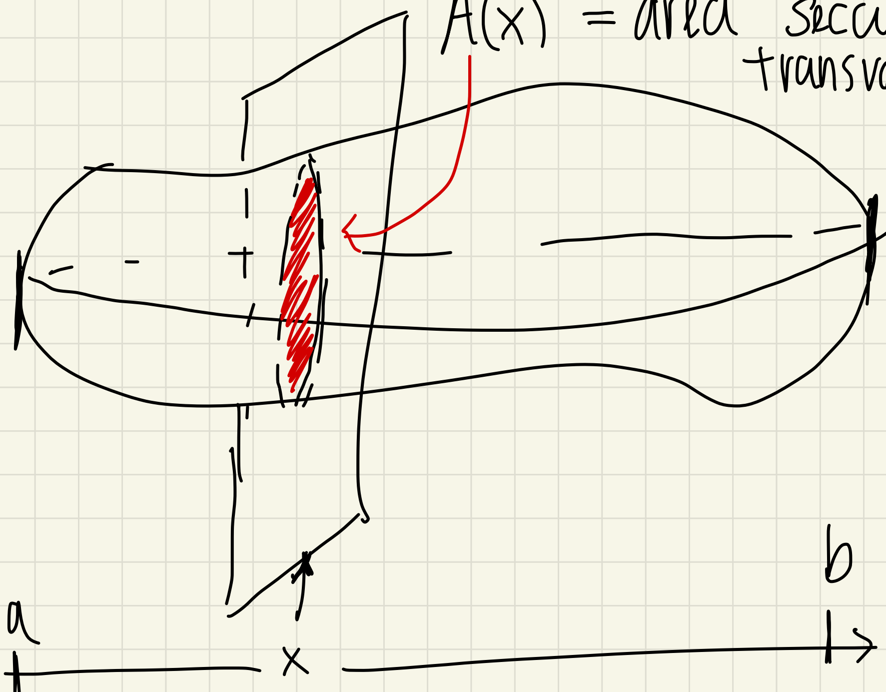
Clase 11: volúmenes

S1: Mier 16/09
Entra hasta volúmenes

Recordemos cómo calcular volúmenes:

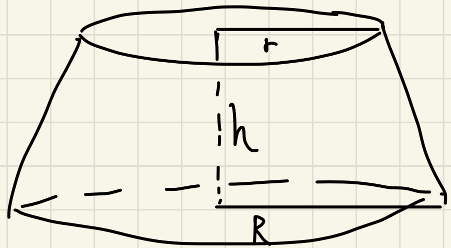
$A(x)$ = área sección transversal

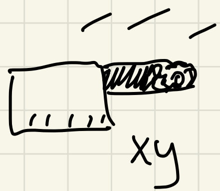


$$\text{Vol} = \int_a^b A(x)\,dx$$

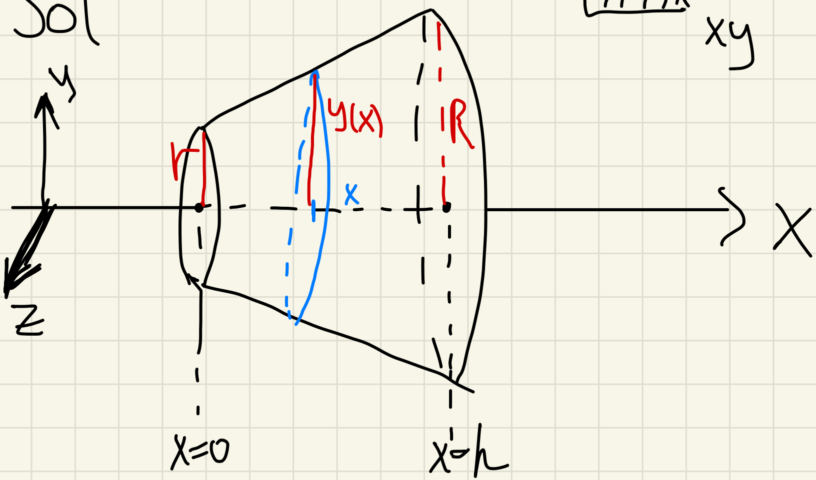
Calcular eso es <u>la</u> dificultad de los problemas

Ej: Calcule el volumen de un cono truncado:



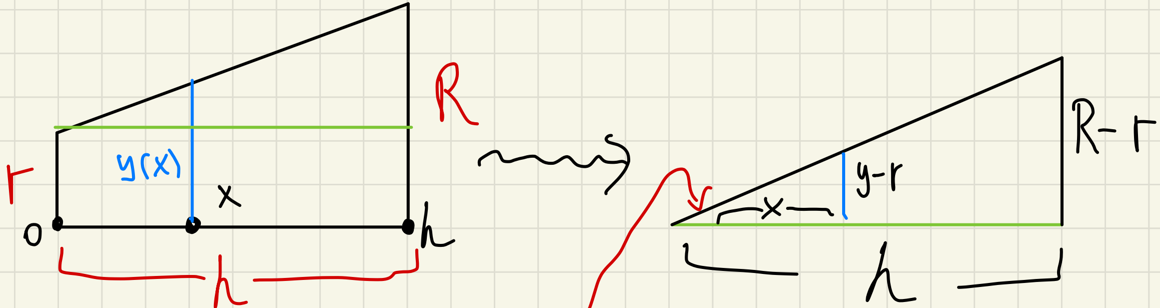


Sol



Pasos

- Necesitamos el Área del circ. azul, $A(x)$.
- Para eso, necesitamos su radio, $y(x)$
- Luego, $\int_0^h A(x)\,dx = vol$



$$\tan(\ ) = \frac{y-r}{x} = \frac{R-r}{h}$$ ← podemos despejar y

$$y-r = \frac{x}{h}(R-r)$$

$$y = \frac{x}{h}(R-r)+r$$

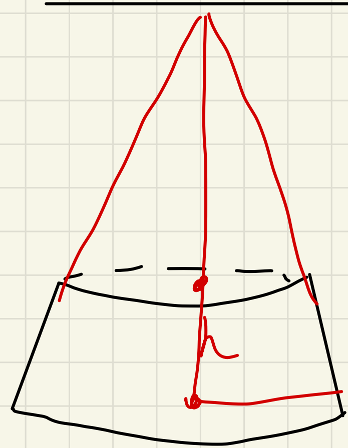
Con esto, podemos calcular el área sección transv.

$$A(x) = \pi y^2 = \pi\left(\frac{x}{h}\cdot(R-r)+r\right)^2$$

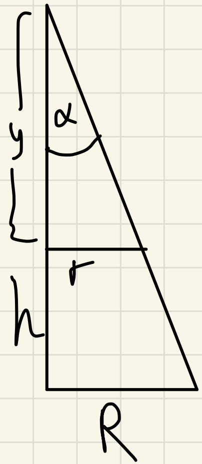
Luego el volumen es

$$\text{vol} = \int_0^h \pi\left(\frac{x}{h}(R-r)+r\right)^2 dx = \frac{1}{3}\pi h\left(r^2+rR+R^2\right)$$

Otra forma:



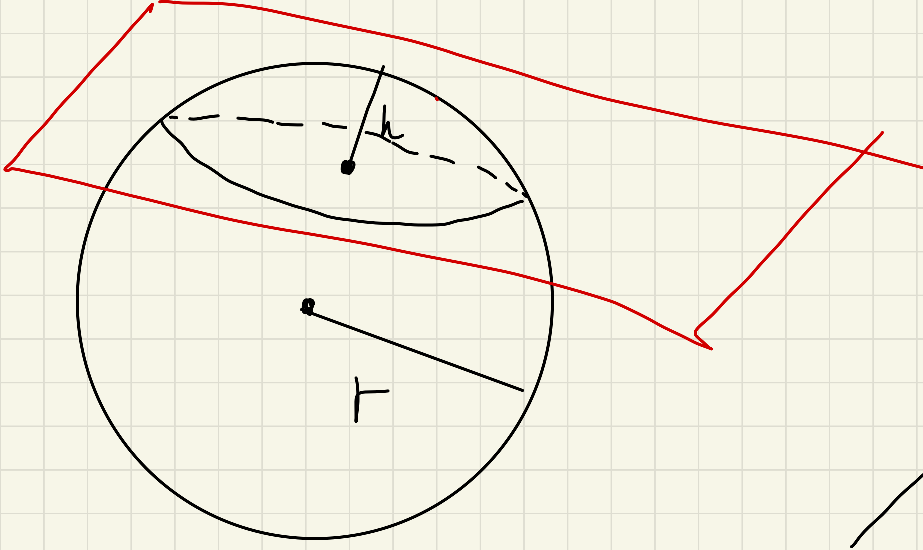
cono truncado = cono grande − cono chico

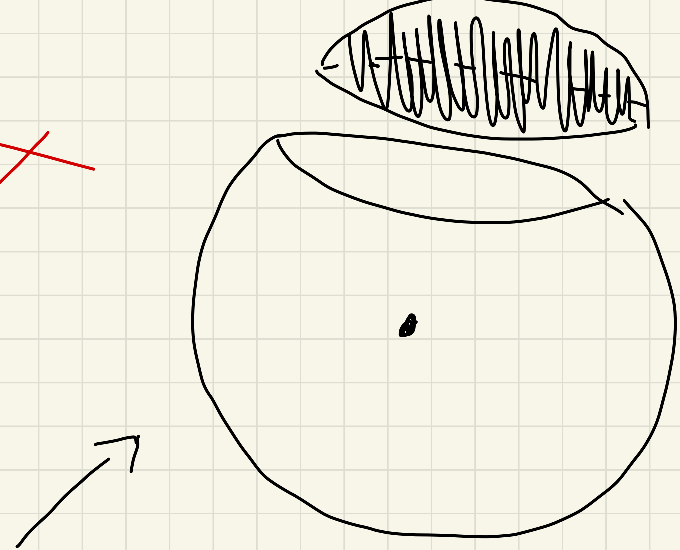


$$\tan\alpha = \frac{r}{y} = \frac{R}{y+h}$$

- despeja y
- Saca Volúmenes
- resta volúmanes
- etc

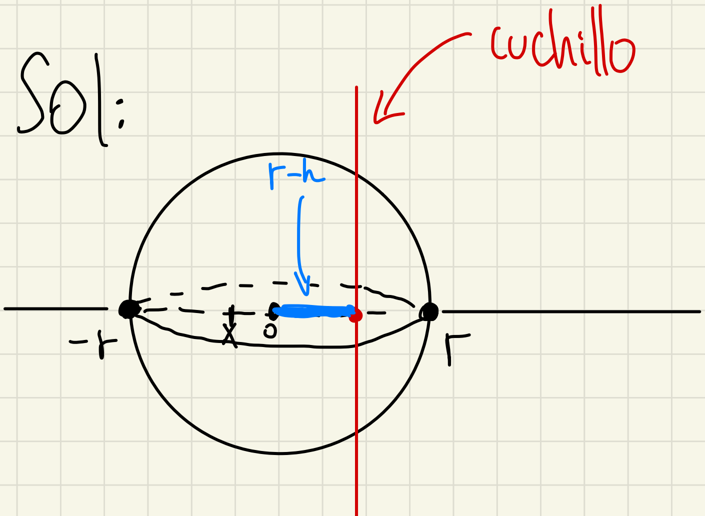
Ej: calcule el voluman del casco esférico:





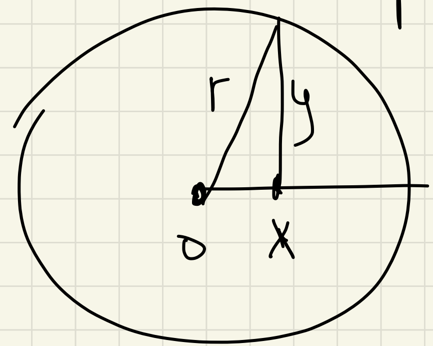
Melón con vino

Sol:



Secciones transversales:
círculos
radio?
área?

$$A(x) = \pi(r^2 - x^2) \longleftarrow \text{clase anterior}$$



$$r^2 = y^2 + x^2$$

$$y^2 = r^2 - x^2$$

$$A(x) = \pi y^2$$

$$= \pi(r^2 - x^2)$$

$$\text{Vol} = \int_{r-h}^{r} A(x)\,dx$$

$$= \int_{r-h}^{r} \pi(r^2 - x^2)\,dx$$

$$= \frac{1}{3}\pi h^2 (3r - h)$$

Para la esfera completa:

$$\int_{-r}^{r} \pi(r^2 - x^2)\,dx$$

^ : elevado

Verificaciones:

- Qué pasa si $h=0$? el casco tiene vol 0

$$\frac{1}{3}\pi h^2(3r-h) \xrightarrow[h=0]{} 0 \quad \checkmark\checkmark$$

- Qué pasa si $h=r$? el casco es la mitad de la esfera ($\text{vol} = \frac{1}{2}\cdot\frac{4}{3}\pi r^3$)

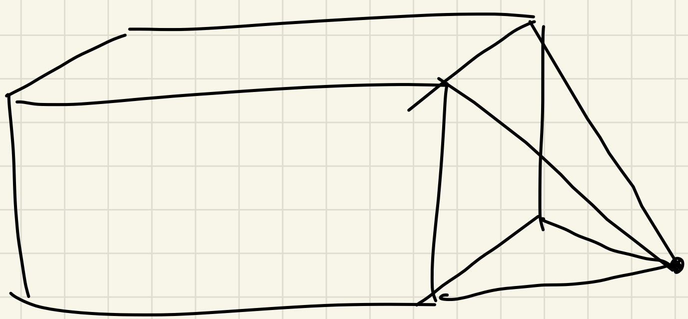
$$\frac{1}{3}\pi h^2(3r-h) \xrightarrow[h=r]{} \frac{1}{3}\pi r^2(3r-r) = \frac{\pi}{3} r^2 \cdot (2r)$$

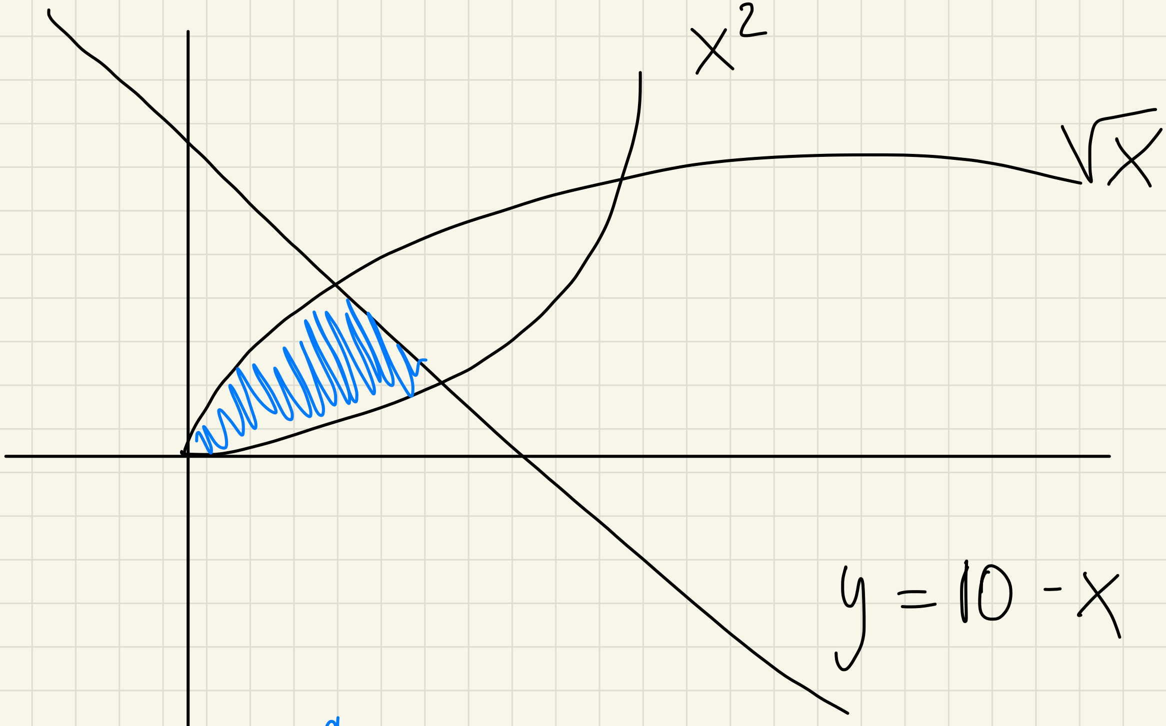
$$= \frac{2\pi r^3}{3}$$

• Que pasa si $h = 2r$? el casco es toda la esfera (vol $= \frac{4}{3}\pi r^3$)

$$\frac{1}{3}\pi h^2(3r-h) \xrightarrow[h=2r]{} \frac{1}{3}\pi (2r)^2(3r-2r)$$

$$= \frac{1}{3}\pi 4r^2 r = \frac{4}{3}\pi r^3$$

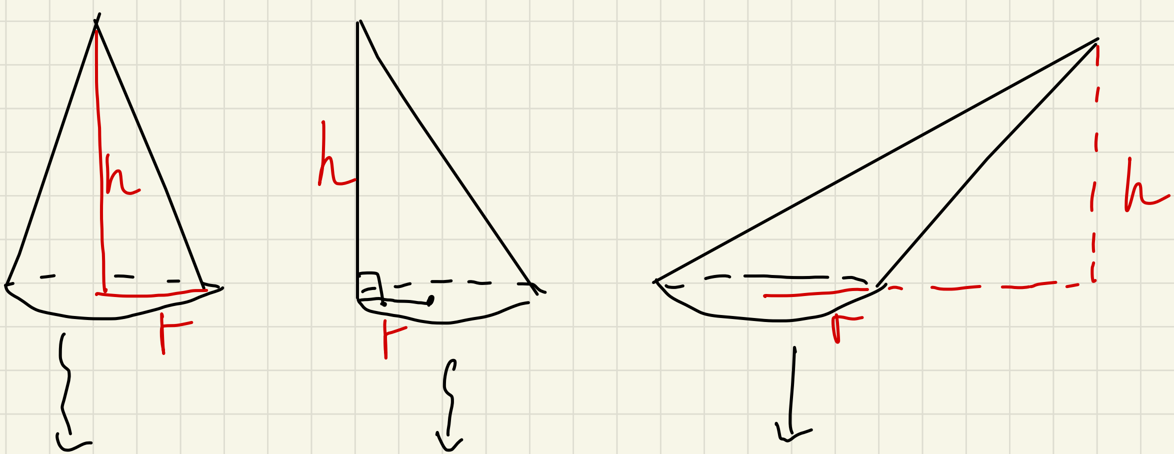




Ej : $\displaystyle\int_{-1}^{1} \frac{\tan x}{etc}\, dx$ ← calculadora

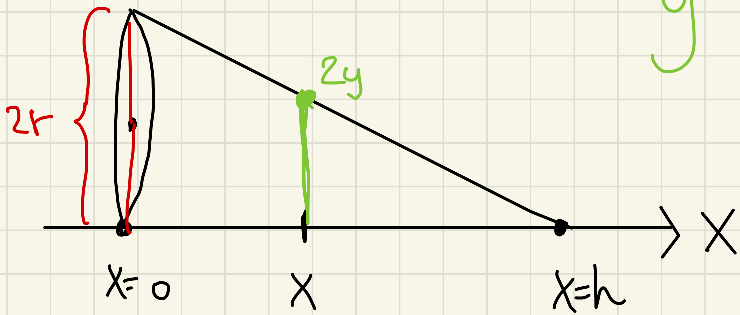
$$\boxed{\frac{d}{dt}\int_{-t^2}^{0}\frac{\cos(x)}{\sqrt{x^2+1}}\cdot\log|x|\,dx}$$

Ej: compruebe que los siguientes sólidos tienen igual volumen

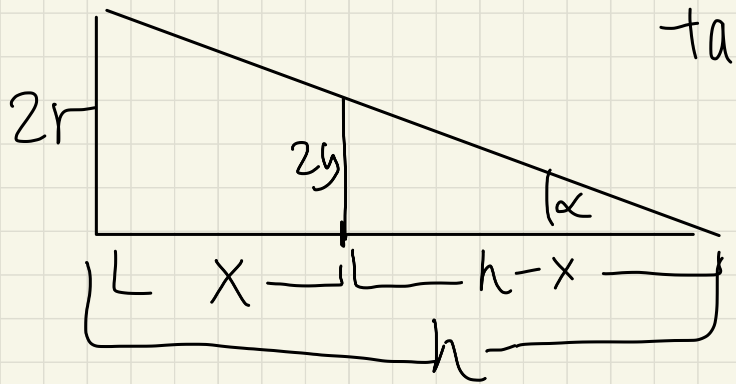


$vol = \frac{1}{3}\pi r^2 h$ $\qquad vol_2 \qquad vol_3$

$\boxed{vol_2}$



y = radio sección transv. (círculo)



$$\tan\alpha = \frac{2y}{h-x} = \frac{2r}{h}$$

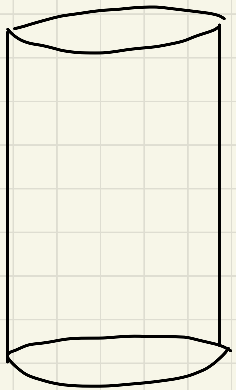
$$y = \frac{r}{h}(h-x)$$

Área sección transversal:

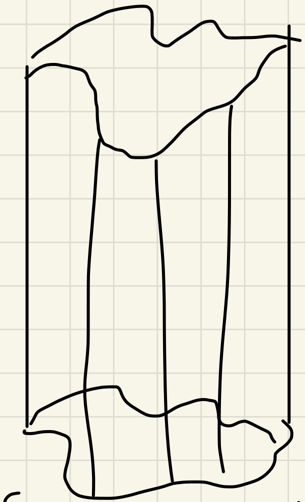
$$A(x) = \pi y^2 = \pi\left(\frac{r}{h}(h-x)\right)^2$$

$$Vol_2 = \int_0^h \pi\left(\frac{r}{h}(h-x)\right)^2 dx = \frac{1}{3}\pi r^2 h$$

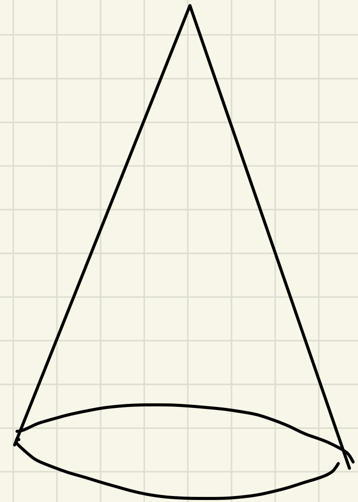
Ej: cuál de las siguientes figuras tiene mayor/menor volumen? por qué?



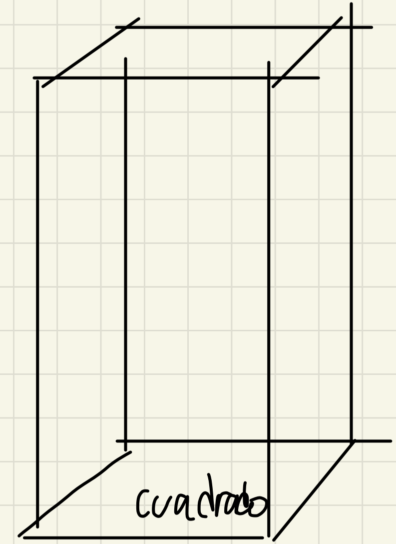
Area base $= A$



Área base $= A$



Área base $= 4A$



lado base $= \frac{1}{2}\sqrt{A}$

Todos con igual altura, h.

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